Report on
E. McCann (1998)
"An Error in Professor Scully's WP14"

by

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1 Background

In a recent Inland Revenue Department Working Paper, Gerald Scully (1996) developed a model of economic growth which featured both private and goverment sectors and sought to find the tax rate that optimised the rate of economic growth. Upon analysis, the model yielded a simple solution for the optimal tax rate in terms of the parameters of the production function. The solution obtained depends on whether increasing or constant returns to scale are present in the production function. An empirical analysis was conducted using least squares regression of growth rates on functions of tax rates and empirical estimates of the growth rate optimising value of the tax rate were calculated and found to be in the vicinity of 20% for the New Zealand economy over the period 1927-1994.

The working paper was critiqued by Ewen McCann in two notes (1998a&b). The first note (1998a) made the following points:

1. The definition of the growth rate g and the production function were claimed to be inconsistent, leading to a contradiction in the formulation of the model

2. The formula for the growth optimising value of the tax rate $\tau^* = \frac{b}{b+c}$ was claimed to be incorrect in view of the claimed contradiction between g and the production function.

3. The statistical test of the constant returns to scale restriction $b + c = 1$ was asserted "to fail because it requires the invalid unrestricted form of equation (8)".

4. In sum, the empirical work was claimed to be unsubstantiated because "the conclusions of the paper are drawn from an estimate of an invalid equation".

This critique used the assumption that the growth rate was constant over adjacent periods. In a second critique, McCann (1998b) allowed for variable growth rates in his analysis of WP14. This critique made the additional points:

1-b. "The model does not have a long run equilibrium outside the constant returns to scale situation".

2-b. "The econometric test used in WP14 to determine whether or not $b+c=1$, fails ... because it requires an estimate using unrestricted values for $b+c$, ... a situation we have now shown to be contradictory in the steady state."

3-b. In a 'disequilibrium' situation $g_t \neq g_{t-1}$, yet use of $b+c=1$ implies $1+g_t = 1+g_{t-1}$ and it is claimed that "the simultaneous use of these two conditions is a contradiction".

4-b. It is claimed in conclusion that "the flaws in the mathematics of WP14 mean that the paper does not demonstrate that the GDP growth maximising tax/GDP ratio is about 20%."

2 Analysis of the Scully Model and Empirical Tests

2.1 The role of constant returns to scale The model in Scully (1996) is a simple growth model relying on the production technology

$$Y_t = aG_{t-1}^b \left[(1-\tau) Y_{t-1}\right]^c = a\left(\tau Y_{t-1}\right)^b \left[(1-\tau) Y_{t-1}\right]^c = a\tau^b (1-\tau)^c Y_{t-1}^{b+c}. \tag{1}$$

This model has an equilibrium growth path solution of the form $Y_t = (1+g)^t Y_0$, with initial condition Y_0 at $t=0$ and equilibrium growth rate g, provided

$$(1+g)^t Y_0 = a\tau^b (1-\tau)^c \left[(1+g)^{t-1} Y_0\right]^{b+c} \tag{2}$$

i.e. if

$$(1+g) Y_0 = a\tau^b (1-\tau)^c Y_0^{b+c}$$

and

$$(1+g)^{t-1} = (1+g)^{(t-1)(b+c)} \quad \text{for all } t \geq 1$$

These conditions require that

$$b+c=1, \tag{3}$$

or constant returns to scale in the production technology. Hence, (1-b) of McCann is correct.

2.2 The Equilibrium Growth Path Manifold and the Growth Optimising Tax Ratio Since the production technology is parameterised by the tax ratio τ, the implied equilibrium growth rate is also parameterised by τ. In consequence, there is an equilibrium manifold determined by the dependence $g = g(\tau)$. From (2) and (3), we have

$$1 + g = a\tau^b (1-\tau)^c = a\tau^b (1-\tau)^{1-b} \tag{4}$$

so that $g(\tau) = a\tau^b (1-\tau)^c - 1$, and optimisation of g with respect to τ leads to

$$\tau = \frac{b}{b+c} = b \tag{5}$$

under (3). This concurs with Scully's (1996) formula (8′) for the optimal tax ratio τ^* in the constant returns case. Thus, Scully is correct in this case.

2.3 The Empirical Estimates The empirical estimates of the growth maximising tax ratio in Scully (1996) are based on least squares regressions of growth rates on functions of the tax ratio. Several empirical estimates are reported. These cover the following regression formulations:

$$1 + g_t = \alpha_1 + \beta_1 \tau_t + \gamma \tau_t^2 + \text{error}, \tag{6}$$

and

$$\log\left(\frac{1+g_t}{1-\tau_t}\right) = \alpha_2 + \beta_2 \log\left(\frac{\tau_t}{1-\tau_t}\right) + \text{error}. \tag{7}$$

The formulation of equation (6) is based on the observed nonlinear relationship between the tax ratio and growth rates reported in Scully's Fig. 2. Direct optimisation of the systematic relation

$$1 + g_t = \alpha_1 + \beta_1 \tau_t + \gamma \tau_t^2$$

from this equation leads to an optimising tax ratio of $\tau_t^{**} = -\frac{\beta_1}{2\gamma_1}$ and estimation of this equation by least squares leads to $\widehat{\tau}_t^{**} = 0.212$.

Equation (7) follows directly from the form of the equilibrium manifold (4) and implicitly assumes that the economy is on the equilibrium manifold up to an additive error. Estimation of this equation leads to $\widehat{\tau}_t^* = 0.202$, or 0.197, depending on whether the CPI or implicit GDP deflator is used to obtain the real output measure.

2.4 Econometric Issues The main econometric issues that arise in this empirical work are the following:

E1. Empirical estimation of (6) and (7) implicitly assume that the tax ratio τ_t is an exogenous variable. This assumption is unlikely to be satisfied empirically. It is not mentioned in the paper and is not justified there. It could be subjected to an empirical test, but this would require instrumentation.

E2. The construction of the GDP growth series relies on 9 year moving averages and therefore reflects the effects of private and public spending over several years. It therefore seems more realistic to formulate an econometric model for this data set with distributed effects of tax rates on growth over time.

E3. The formulation of the production function (1) omits technological progress and infrastructure effects of taxation. In the New Zealand case, especially in view of the importance of the welfare state over much of the sample period and ongoing public spending on education and health, formulations such as (6) and (7) seem rather inadequate and omit several important channels of taxation effects on economic growth, primarily in terms of infrastructure spending and human capital investment expenditures by the public sector. It might be argued that equations (6) and (7) measure only 'reduced form' effects and therefore would incorporate such channels of influence in an indirect way, even though they do not enter the production technology on which (7) is based. However, since the tax ratio is not exogenous and exogeneity is not tested, it cannot be argued that these equations are reduced forms. In consequence, there is a need for a more sophisticated structural model that allows the production technology to incorporate the effects of infrastructure and human capital expenditure. It would also be prudent to carry out tests for omitted variable bias in equations (6) and (7).

E4. While standard errors are given for the coefficients in (6) and (7), no standard errors for the optimising value of the tax ratio, $\widehat{\tau}^*$, are given. It would be prudent to provide these, because they would give the range of values for the growth optimising tax ratio that are supported by this data set and empirical methodology and would show some of the implied uncertainty about the conclusions that are drawn from this empirical work. Ideally, one should obtain robust estimates of these standard errors that allow for some equation misspecification. The same applies to the reported standard errors of the coefficients. Some more extensive tests of potential equation misspecifiation, other than DW statistics would be helpful here.

E5. Some sensitivity analyses to assess the effects of the growth rate averaging that was used here should be conducted.

E6. The empirical regressions seem to rely in an important way on data in the 1920's and early 1930's. In particular, all of the principal observations of low tax rates seem to rely on data for the first part of the sample. Indeed, the hypothesised quadratic structure of the relation between growth and tax ratios seems to be heavily dependent on the inclusion of this early part of the sample in the regression. During this early part of the sample period, the scale of the government sector grew substantially and with it the provision of infrastructure and human capital from which substantial early gains in average growth set in. In consequence, there seem to be major regime differences between the early period and later periods of the data and the empirical regression estimates may be quite dependent on the extent to which this period is covered in the sample data. It would be prudent to study whether such dependence plays a role in the findings.

E7. Extension of the estimates to include more recent data than 1994 also seem desirable, as recent growth has been strong. However, allowance for the 9 year moving average effects again seem to be important, because recent gains would be attenuated from averaging over the business cycle and the recession of the early 1990's, in particular.

E8. Regarding the matter of the appropriate test of constant returns to scale, it is not unusual to formulate a regression version of the model that relaxes the restriction and conduct t- tests such as those reported in Scully (1996, p.8). These can be justified as Wald tests, and provided the null hypothesis is correct and the model is not misspecified in other ways, the tests will be valid. In the present case, it would have been preferable to conduct robust tests that allowed for some misspecification in the maintained equation.

3 A Pilot Fragility Analysis

As indicated in E6 above, the early part of the sample period looks to be quite influential in the empirical work. A pilot analysis was conducted to determine the extent of the influence. The results given here are purely illustrative of the effects involved. We do not purport to suggest that the outcome of these regressions be used in the empirical debate, our position being as stated above - that a more detailed econometric study be undertaken paying attention to all of the issues raised there.

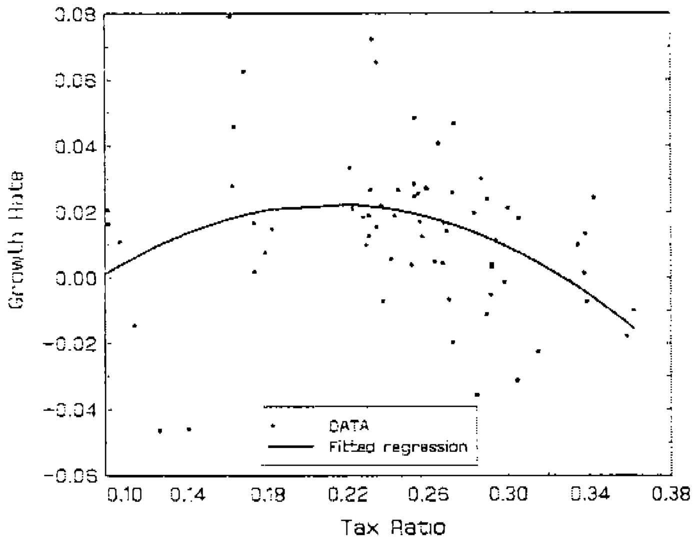

Figure 1: Growth Rate Regression: 1927-1994

Regressions of the form (6) were run from initial conditions starting at $t_0 \in [1927, 1933]$. The data were not prefiltered by taking moving averages. The results for 1927 and 1933 are given below and Fig.'s 1 and 2 graph the data against the fitted quadratic regression line. The least squares regression results for the data from 1927 – 1994 are as follows:

$$\begin{array}{rcccccccc} g_t = & -0.053 & + & 0.71\tau_t & - & 1.68\tau_t^2, & R^2 & = & 0.14 \\ & (0.036) & & (0.27) & & (0.52) & & & \end{array} \quad (8)$$

where the standard errors given in parentheses are robust to serial dependence and heteroscedasticity. The coefficients in this regression lead to a point estimate of the growth optimising tax rate of $\tau^* = 0.21$, which is precisely the same as the point estimate reported in Scully (1996, p.2) for a similar regression using the raw data. The results for the data over the period 1933-1994 are:

$$\begin{array}{llllll} g_t = & 0.068 & - \; 0.17\tau_t & - \; 0.09\tau_t^2, & R^2 & = \; 0.24 \\ & (0.043) & (0.33) & (0.62) & & \end{array} \tag{9}$$

again with robust standard errors reported. This regression produces a point estimate of the growth optimising tax rate of $\tau^* = -0.90$.

The outcome is quite apparent. The initial observations over 1927-1932 play a vital role in the shape of the fitted regression function and the value of the optimising tax ratio. In fact, the coefficients seem poorly determined and sensitive to a few points of data.

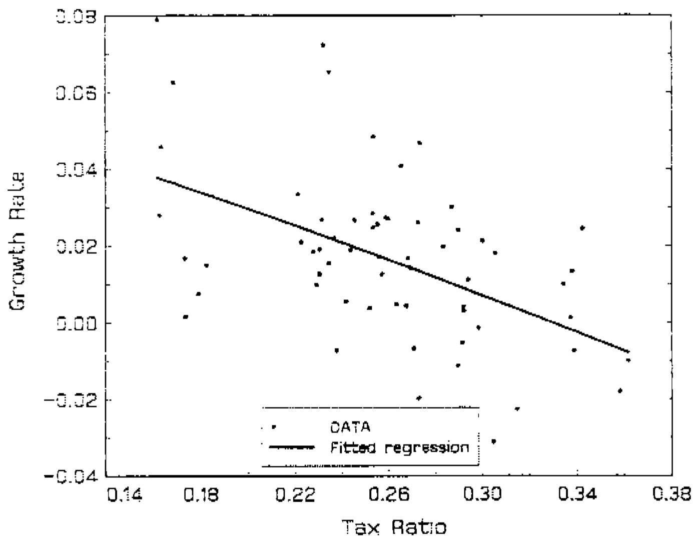

Figure 2: Growth Rate Regression: 1933-1994

One would expect this sensitivity to initialisation to persist in the case of the model-based regression (7) and this should be checked. To do so, one would need the time averaged data used in Scully (1996) or have the procedure for dealing with end points in the time averaging described, so that the results could be replicated.

4 Conclusions

1. The imposition of constant returns to scale is important to the internal consistency of the Scully model and helps in formulating the notion of an equilibrium growth manifold, as shown above in Section 2.2. This manifold is an important feature in the use of the model for the analysis of the optimal tax ratio.

2. The optimal tax ratio (5) should be interpreted as referring to a class of alternate growth paths in steady state.

3. In the light of the above, I concur with McCann (1998a & b) that there are some technical deficiencies in the development of the model in Scully (1996). However, I do not see these particular deficiencies as fatal, nor do I think they are the most important potential flaws in the investigation of the issue of growth optimising tax rates, optimal government size, and deadweight loss estimation. I see all of (E1) - (E7) as more important considerations.

4. The use of an equilibrium growth model to analyse optimal tax rates may be convenient, but it is not clear that it is empirically relevant. Notwithstanding this remark, the regression (6) can be motivated by empirical considerations alone, and this regression certainly gives estimates (8) over the full sample period that conform to the later results based on the theory-model formulation and time-averaged data. The problem, as revealed in the subsequent regression (9), is that the results are fragile to a change in initialisation.

5. A preferable theoretical approach that seems more suited to the empirical conditions would have been to construct a cyclical growth or disequilibrium growth model, for which a particular solution was the equilibrium manifold studied in Scully's paper. In this respect, I agree with the drift of McCann (1998b). If this approach had been adopted, then the stability of the system could have been analysed and some

well designed simulations may have heiped to shed light on the time paths of general solutions to the steady state.

6. The issues raised in this research have major policy implications. It would seem important to check the robustness of the findings to a broader maintained hypothesis and to see how sensitive the conclusions are to certain aspects of the model formulation, data construction, and econometric methodology. The empirical estimates in Section 3 give some idea of the fragility of these regression results. I recommend that this line of investigation be pursued further.

7. Regarding the matter of empirical testing for constant returns in Scully (1996), I see no special difficulty here. The main problem arising from these tests is that they are not robust against misspecification.

References

McCann, E. (1998a). "An Error in Professor Scully's WP14", IRD mimeographed, 17 April, 1998.

McCann, E. (1998b). "An Error in Professor Scully's WP14", IRD mimeographed, 11 May, 1998.

Scully, G. (1996). "Taxation and Economic Growth in New Zealand", IRD Working Paper, 14.